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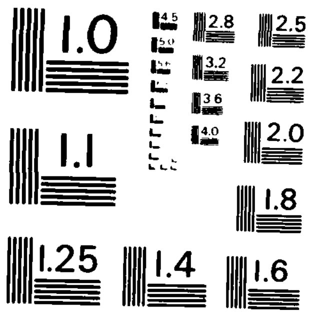

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: AFOSR-TR- 83-0661	2. GOVT ACCESSION NO.: AD-A131 316
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Approximation Methods in Multidimensional Filter Design and Related Problems Encountered in Multidimensional System Design	5. TYPE OF REPORT & PERIOD COVERED: Final (1-1-78 to 1-31-83)
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Professor N. K. Bose	8. CONTRACT OR GRANT NUMBER(s): AFOSR 78-3542
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of Pittsburgh, Department of Electrical Engineering, Pittsburgh, PA 15261	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 611C2 F / 2304/A6
11. CONTROLLING OFFICE NAME AND ADDRESS: AFOSR/NM, Division of Mathematical & Information Sciences, Bolling AFB, D.C. 20332	12. REPORT DATE: 3-21-83
	13. NUMBER OF PAGES: 34
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release - Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multidimensional Systems	Spatio-Temporal Filtering
Rational Approximants	Primitive Factorization
Stability	Irreducible polynomials
Stabilization	Matrix Stieltjes series
Robustness	Digital Filters over finite field

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research conducted contributes towards the development of a theory to analyze and design linear shift-invariant multivariable multidimensional discrete and continuous systems. Recursive schemes to compute rational approximants to a power series in two variables having constant matrices for coefficients are developed. Approximants to special matrix power series are investigated and the properties of these approximants are delineated in a strictly mathematical setting and their implications are interpreted via

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) physical reasonings. Algebraic procedures to test the approximants for stability are provided, and criteria for guaranteeing the invariance of properties like positivity and stability under parameter changes or perturbation are advanced. Attention is directed throughout towards the reduction of algebraic computational complexity. In the important problem of filter stabilization without appreciable change in the magnitude of the frequency response, recent results on multiplicative computational complexity theory is exploited to demonstrate the feasibility of implementing efficiently a 2-D discrete Hilbert transform. Criteria for 2-D rational approximants to be maximally flat are obtained.

The structure and properties of impulse response arrays for discrete-space systems over a finite field of coefficients are investigated. A complete characterization of arrays whose properties are very close to the properties of pseudorandom arrays, is given; instead of two distinct levels of the discrete autocorrelation function, these characterized arrays are shown to have a maximum of three distinct levels, whose values can be explicitly calculated from the denominator polynomial of the transfer function generating the array.

The feasibility of obtaining the primitive factorization of bivariate polynomial matrices whose coefficients belong to an arbitrary but fixed field is demonstrated by the development of an algorithm whose implementation require operations over the specified field and which also produces factors with coefficients over the same field and not over some extension field. This result opens the door to several challenging practical problems in the realization theory of multidimensional systems. The primitive factorization result is also used in the technique developed to stabilize 2-D discrete-space plants using causal and weakly causal compensators.

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b. Research Objectives

(1) Rational Approximants

The topics here will be presented in a concise but clear manner so that the reader encounters little difficulty in understanding the context and technical merits. Most specifications are unrealizable from the physical realizability standpoint and have to be approximated with respect to a chosen error criterion. The approximation may be done either in the frequency or spatial domains. It was recognized that the quality of the approximant must be coupled to the facility or convenience with which the approximant can be calculated. This necessitated the demand for attention to the scopes for development or exploitation of fast algorithms; that is, an understanding of the problem structure was considered crucial. At the same time, the approximation technique was required to be fundamental and broad enough to support a wide variety of applications. It was decided that the Padé approximation technique would serve as an ideal nucleus from which the desired research directions and objectives could ideally be reached.

The Pade approximant of order [L/M] to the formal power series $t(z) = \sum_{k=0}^{\infty} t_k z^k$ is the rational function $a_L(z)/b(z)$, where $a_L(z)$ and $b_M(z)$ are polynomials of degree at most L and M respesctively such that

$$t(z) - \frac{a_L(z)}{b_M(z)} = 0(z^{L+M+1}) \qquad (1)$$

and $0(z^{L+M+1})$ in (1) is taken to mean that terms of any order less than z^{L+M+1} are missing. An approximant of any order, if it exists, is unique and the problem of computing the approximant essentially consists of solving a system of simultaneous linear equations

$$H_M [b_M \ b_{M-1} \ \cdots \ b_1]^t = -[t_{L+1} \ t_{L+2} \ \cdots \ t_{L+M}]^t \qquad (2)$$

where, H_M is a (M x M) Hankel matrix obtained by setting $[t_{L-M+1} \ t_{L-M+2} \cdots t_L]$ as the first row and $[t_L \ t_{L+1} \cdots t_{L+M-1}]^t$ as the last column,

b_k's in (2) are coefficients of polynomial $b_M(z)$ and 't' denotes the transpose of a matrix. It is well known, however, that equations of the type in (2) occur in the problem of partial realization of systems from prescribed input-output maps [1]. Recursive $O(n^2)$ algorithms which exploit the Hankel structure of the matrix H_M, to solve for the system of equations (2) have been proposed [2]. It has also been noted recently [3] that these recursions fall under the class of recursionsknown as Lanczos recursions [4]. Furthermore, the same recurrence formula occurs in the entirely different context of shift-register synthesis and is called the Berlekamp-Massey relation [5]. One objective of the research was to develop a scheme for recursive computation of the Padé approximants, when the coefficients of the power series t(z) are rectangular matrices.

A closer examination of the recurrence relations (in the scalar case) reveals that the denominators of the successive members of the sequence of approximants are connected by a three term recurrence formula similar to the one followed by polynomials, which are orthogonal on a real interval. In fact, it has been shown that, under some restrictive hypothesis, the sequence of polynomials $\{z^M b_M(z^{-1})\}_{M=0}^{\infty}$ associated with the denominators of the sequence $\{[(M-k)/M]\}_{M=0}^{\infty}$ indeed form a set of orthogonal polynomials of the type just mentioned. Furthermore, the sequence of successive convergents of a continued fraction expansion of a special type associated with t(z) is known to be the same as the sequence of Padé approximants: [0/1], [1/0], [1/1], [2/1], [2/2], . . . to t(z).

Also, the infinite series t(z) is known to be a Stieltjes series if certain positive definiteness conditions on the Hankel matrices H_M in (2) are satisfied. A result of significant interest in the context of this discussion is that the sequence of Padé approximants of certain orders to a Stieltjes series can be shown to be realizable driving point impedance functions of electrical networks consisting of two types (RC, RL, or LC) of elements. Proof for this fact can be given by classical ladder type synthesis techniques or via a consideration of the Cauchy index of rational functions of one variable.

Another objective of the research was to investigate into the properties of the "denominator" polynomial matrices with respect to any recurrence relation they might obey. The Stieltjes series was planned for study in a matrix setting, especially with the objective of exploring into possible links of the approximants to multiport network theory. The theory of matrix polynomials, orthogonal over a real interval, was then natural to investigate, especially with the possible availability of any physical insight from network theory.

In view of the established importance of 2-D systems theory from both theoretical and applications standpoints, the next objective was to attempt to extend the Padé approximation scheme to the 2-D case by requiring the rational approximant of prescribed degree $\hat{t}(z_1,z_2)$ to be such that R_{ij} in (3) is zero over a

$$\hat{t}(z_1, z_2) - t(z_1, z_2) = \sum_0^\infty \sum_0^\infty R_{ij} z_1^i z_2^j \qquad (3)$$

finite set of points (i,j) having a chosen geometry in the two-dimensional lattice. Two different geometries have been considered in [6] and [7]. Numerous other possibilities, in this direction of generalization exists with respect to choice of geometry of these lattice points and it is to be expected that the approximants resulting from considerations of various geometries will have properties desirable in the synthesis of two-dimensional systems. An important objective of the research was to investigate into the structure and properties of these 2-D rational approximants, in addition to providing fast methods for their computation.

The approximants are required to satisfy certain desirable properties, when used to design complex systems. Two such properties are stability and robustness to parameter variations. Another objective of the research was to provide suitable means for incorporating these desirable features in the approximants.

The final objective in the approximation phase of the research was to investigate into the conditions on the coefficients of the rational approximants so that certain desirable properties like maximal flatness or equiripple behavior in the frequency response of the designed multidimensional filters would be obtained.

ii. Multidimensional Signal Processing Over a Finite Field

The theory for 1-D discrete systems which are linear over the finite field of two elements (binary field), GF(2), is well established. One-dimensional discrete systems which are linear with respect to the finite field of two elements, 0 and 1, have been developed for applications such as generation of pseudorandom sequence and error-correcting codes. Such systems are characterized by rational functions and polynomials in one variable with coefficients over GF(2). Recently, the need for study of properties of two variable rational functions in the quotient field of the ring of bivariate polynomials with coefficients belonging to GF(q), has been felt in the synthesis of psuedornadom noise arrays that exhibit special correlation properties, system identification, and two dimensional filtering over a finite field. There is no satisfactory definition for primitivity of 2-D polynomials over a finite field, though that concept has played a vital role in the 1-D case especially in error-correcting code construction and generation of maximum length shift register sequences, which in turn are ideally suited for the identification of linear systems using cross-correlation techniques. For bivariate polynomials, the study of primitivity, the generation and enumeration of irreducible polynomials, the study of their corresponding array and their autocorrelation properties was, therefore, considered to be potentially very useful. An added advantage to this study, is the observation of the fact that there is much in common between the algebraic structure that characterize these and the approximation problem discussed earlier.

iii. Primitive Factorization, Coprimeness, Matrix Fraction Description

Research in this area was considered to be very fundamental to the general theme of research conducted.

The need for factoring a given polynomial matrix $A(z)$ in the form $A(z) = A_1(z)\ A_2(z)$ (where the polynomial matrix factors $A_1(z)$ and $A_2(z)$, besides having elements in the same ring to which the elements of $A(z)$ belong, might also be required to satisfy certain special properties), occurs in many situations. These include network theory, filtering of multiple time series, system theory, damped vibrations, transport theory, and Wiener-Hopf equations. Special attention to spectral factorizations, where $A_1(z)$ and $A_2(z)$ have their spectra located one inside and the other outside a Cauchy contour with $A(z)$ analytic on a neighborhood of the contour, has been given in [8]. A more general type of factorization theory occurs in the polynomial matrix approach to realization theory of linear systems [9], which, incidentally, has close links to the state-space realization theory [10]. A state-space realization, $S(z) = H(zI-F)^{-1}G$ of a matrix of strictly proper rational functions has minimal dimension if and only if (F,G) is controllable and (F,H) is observable. On the other hand, a polynomial realization,

$$S(z) = W(z) + V(z)\ [T(z)]^{-1}\ U(z)$$

is irreducible if $T(z)$, $U(z)$ are relatively left prime and $T(z)$, $V(z)$ are relatively right prime. The connection is provided by a theorem of Rosenbrock which states that a pair of constant matrices (F,G) is controllable if and only if the pair $zI - F$, G of polynomial matrices is relatively left prime.

Recent developments in multidimensional systems theory [11], [12], motivated by a variety of practical applications [13] necessitated investigations of the scopes for extension of the univariate (1-D) multivariable (or multi-input multi-output) results of Rosenbrock and others to the n-D ($n > 1$) case. The realization theory based on the 1-D polynomial matrix approach is valid for polynomial matrices (generated, for example, from an initial non-irreducible realization of $S(z)$) with elements from an arbitrary principal ideal domain and when the solution (say, an irreducible realization) is to be constructed, the domain is further required to be Euclidean. In n-D ($n > 1$) system theory, this is not the case. In [14], Morf, Levy and Kung showed how the greatest common right divisor (gcrd) could be extracted from two specified bivariate polynomial matrices based on their primitive factorization theorem. The operations necessary to implement the extraction scheme, however, required the ground field to be algebraically closed. Subsequently, Youla and Gnavi [15] showed the infeasibility, in general, of primitive factorization for polynomial matrices in three variables and among other interesting results also presented a primitive factorization algorithm for 2-D polynomial matrices similar to the one in [7]. This algorithm also requires the ground field to be algebraically closed. This requirement is not satisfied in many problemsof practical interest. In continuous linear system theory, the ground field is usually real, while in the discrete case it could also be finite [13, ch. 6]. With the objective of increasing considerably the scope for application of the important theoretical results in [14], [15], the question as to whether or not the primitive factorization theorem, presented in [14,], holds over any arbitrary, but fixed, field of coefficients was posed in [13]. A very important objective of the research was to obtain an answer to the question posed and explore the impact of the result to various applications of interest to the Air Force.

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c. Status of the Research

(i) Rational Approximants

Let K[z] be the set of formal power series over a ring K with indeterminate z. In the present context, K will be the ring of constant rectangular matrices of order m x n, whose elements do not necessarily commute. The matrix Padé approximation problem then consists of finding two matrix polynomials $A_L(z)$ and $B_M(z)$ such that for a given $T(z) \varepsilon K[z]$:

(1) $\delta[A_L(z)] \leq L$, $\delta[B_M(z)] \leq M$, where "δ" denotes "degree"

(2) the residual,

$R(z) = T(z)\, B_M(z) - A_L(z)$

satisfies

$\mathrm{ord}\, R(z) \geq M + L + 1$

In the conventional scalar Padé theory, for $A_L(z)$ and $B_M(z)$ to be devoid of a common divisor of degree greater than zero, it is necessary that $B_M(0)$ be nonzero. In the case considered here, $B_M(0)$ is taken to be nonsingular and without loss of generality it may be set equal to the identity matrix I of appropriate order.

$$B_M(0) = I$$

$A_L(z)\, B_M^{-1}(z)$ will then denote the right matrix Padé approximant of order [L/M]. The LMPA (L stands for left) may be (RMPA) analogously defined. In [c.1], it has been established that (except possibly in certain derogatory cases), a recurrence relation relates the [L+1/M+1], [L/M], and [L-1/M-1] order matrix Pade approximants (left or right), the existence of which is guaranteed from the vailidity of a certain rank condition on the corresponding set of characterizing matrices possessing a block-Hankel structure.

The preceding results have been used to define and study a special type of power series belonging to the ring of constant square matrices. The main result documented in [c.2] is the proof of the relationship between rational approximants of appropriate orders to a specified symmetric matrix power series of the special type referred to and mulitport network synthesis using the matrix version of the classical continued fraction expansion theory and the recently developed artifice of matrix Cauchy index [c.3]. The "denominator" polynomial matrices of the approximants are shown to form an orthogonal polynomial matrix sequence over a real semi-infinite interval. Though mathematical derivations of the properties of the polynomial matrices belonging to the orthogonal polynomial matrix sequence have been given, the reader has also been alerted to the feasibility of network-theoretic justification of the results. It is hoped that these links between mathematical results and the theory of network realizability will kindle interest for further research among scientists coming from either disciplines.

The mathematical literature [c.4], [c.5], contains some results on the extension of the Padé approximation scheme to power series in more than one indeterminate, primarily with a veiw towards applications in problems of physics and numerical analysis. In these approaches, however, the Hankel nature of the characterizing matrix is lost and, therefore, the feasibility of recursive computation is seriously impaired. In the contexts of two-dimensional digital filtering, stochastic realization and image processing, the speed of computation is crucial. In order to conform to this requirement, the questions of existence, nonuniqueness alongwith scopes for recursive computation when existence is guaranteed, are investigated in depth in [c.6]. In [c.6], attention is directed to the exploitation of a block Hankel-Hankel matrix structure (where the elements of the block may also be viewed as block-Hankel matrices) that can be used to characterize the problem. A sequence of vertical and horizontal Padé approximants over the chosen "Hankel matching grid" are defined.

A three-term recurrence formula is developed to compute, first, all the vertical approximants of a prescribed order, in the sequence. In physical problems, the rational approximants are required to be stable. A sufficient condition for BIBO stability based on the zero distribution of a bivariate polynomial, obtained directly via computation of the determinant of a matrix formed from the input data, augmented by a row of matrix indeterminates, has been obtained in [c.7]. Thus, if one is willing to invest one's time in the computation of the determinant of a specified matrix followed by a check for stability of one bivariate polynomial (the determinant computed), the effort required to actually calculate an approximant, which will turn out to be unstable, may be saved. For generalization of the concept of Padé approximants to nonlinear operators and its scopes for tackling nonlinear systems of equations, nonlinear initial and boundary value problems, nonlinear partial differential equations and nonlinear integral equations, a recent dissertation [c.8] is of interest.

Since 1-D techniques for computing Padé approximants are more well developed and naturally, faster, the feasibility of constructing 2-D rational approximants to match exactly a prescribed set of coefficients in a double power series, by using only the 1-D Padé technique has been explored in [c.9]. The matching grid is not restricted to a rectangular grid though for brevity in exposition this type of grid is focussed upon in [c.9]. There, it is shown that it is possible to obtain a 2-D rational approximant by computing several 1-D Padé approximants over power series coefficients that belong to a field and only one 1-D Pade approximant over power series coefficients that belong to the field of rational functions in 1 indeterminate. The procedure and basic philosophy extends easily to the n-D case, $n > 2$. The speed and flexibility in implementation of the scheme is likely to be useful in various types of filter design problems.

Positivity and Stability

Positivity and stability results, which are fundamental in the area of multidimensional systems design were established on firm foundations. It has already been pointed out that rational approximants are useless for implementation, if unstable. A new stability test for n-D digital filters has been obtained in [c.10]. This test dwells on a conceptually different philosophy than other algebraic tests advanced for multidimensional filters, as noted in [c.11], though the computational advantages in implementation may not be present, in general. The problem of testing a polynomial for zeros on a polydisc distinguished boundary is considered in[c.12]. Such tests are required in testing a linear discrete shift-invariant processor for exponential stability [c.13] and in theuse of ceptral techniques [c.14], the discrete Hilbert transform and homomorphic filtering. The test procedures for stability often dwell on the checking of a multivariate polynomial for local or global positivity. It is shown in [c.15] that certain singular cases occurring in the test of a multivariate polynomial for global positivity need not be considered, leading to a considerable improvement in the efficiency in implementation of the test procedure.

Robustness

In problems originating from physical systems, the multivariate polynomials characterizing some properties of the system under study, like stability or existence of limit cycles, might not have integer coefficients and might even possess coefficients which could take arbitrary values over specified intervals. In such cases, it is useful to know the allowable interval within which the coefficients might fluctuate centered around their respective unperturbed values, so that the test implemented on the polynomial with these unperturbed coefficient values, if found to yield the positivity property will guarantee invariance of the positivity property under perturbation of any or all of the coefficients within the allowable interval. In [c.16], it is first shown how the largest symmetric interval centered around each coefficient of a globally positive univariate polynomial may be determined so that the positivity property is invariant under any perturbation of the coefficients within that interval (or its integer multiple). A characterization theorem for the determination of a similar interval is given for multivariate polynomials and a scheme for finding the interval via recursive computation of resultants, allowing the elimination of at least one indeterminate at each step, is given. This result partially contributers towards the resolution of the multidimensional filter stability problem under coefficient fluctuation in the characterizing denominator polynomial of the filter due to any sources of error -- roundoff, quantization etc. In [c.17], the main goal of obtaining conditions for invariance of stability properties as directly as possible in terms of the original coefficients (just as was done with respect to the global positivity property in [c.16]) was obtained for low degree polynomials. Though, higher degree polynomials are not tackled in [c.17], the results should be useful from the practical standpoint, since often the design of higher order systems is based on the design of low order blocks or prototypes.

Stabilization

Following the derivation of a rational approximant and implementation of the stability test a filter may be found to be unstable. A practical problem encountered, then, is to stabilize the filter without appreciable change in the magnitude of the frequency response. Two approaches towards the attainment of this goal are the planar least squares inverse (PLSI) approach and the discrete Hilbert transform (DHT) approach. In [c.18] it is shown using recent results from algebraic computational complexity theory that the multiplicative complexity of computation of a 2-D DHT is not greater than twice the sum of multiplicative complexities of two 1-D DHT's.

(ii) Multidimensional Signal Processing Over a Finite Field

An important fact exploited in many applications (like the construction of error correcting codes) is the existence of irreducible single variable polynomials of any specified degree whose coefficients belong to the finite field, GF(q) (the binary field case, where q=2, is of greatest practical significance). In [c.19], a nontrivial extension of a known procedure to obtain the number of distinct irreducible monic univarate polynomials of prescribed degree m with coefficients in GF(q) is made with respect to multivariate polynomials. Specifically, a procedure is given to determine the number of distinct irreducible polynomials of prescribed degree in each of the independent variables and having coefficients in GF(q).

In [c.20], an analysis of the structure and properties of the impulse response array of 2-D discrete space systems, characterized by rational functions having coefficients in GF(q), is given. It is shown that such an array exhibits a rwo (column) type of periodicity. This property specializes to the doubly periodic property if and only if the denominator polynomial of the transfer function is separable into a product of univariate polynomials. Arrays with a maximum of three levels in the autocorrelation function are identified and explicit expressions for these levels are given. This result provides the complete characterization of arrays which are remarkably close to the pseudo-random arrays sought in a variety of applications including the study of spectrometric imagers and sound diffusion.

(iii) Primitive Factorization, Coprimeness, Matrix Fraction Description

A problem of importance both from the theoretical and application viewpoints has been solved during the course of this research. In 1977 and 1979, algorithms were presented to extract in some sense the content of a full rank matrix A with entries in the ring $K[z,\omega]$ of bivariate polynomials over some field K. However, the algorithms presented in both cases specify and require the coefficient field K to be algebraically closed - typically the field of complex numbers. It is desirable, from theoretical and computational standpoints to have no such restriction on K; so, for example, one could do the factorization over the real field or even the field of rational numbers, provided the coefficients start out in these field. In [c.21] an algorithm which produces a primitive factorization over an arbitrary field K is presented. Several related results leading to a general factorization theorem are also stated and proved.

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[c.21] J. P. Guiver and N. K. Bose, "Polynomial matrix primitive factorization over arbitrary coefficient field and related results," IEEE Trans. Circuits and Systems, 29, October 1982, pp. 649-657.

d. Publications in Technical Journals

1. N. K. Bose and S. Basu, "Tests for polynomial zeros on a polydisc distinguished boundary," IEEE Trans. on Circuits and Systems, vol. 25, Sept. 1978 (Special Issue on Mathematical Foundations of Systems Theory), pp. 684-693.

2. N. K. Bose, "Implementation of a new stability test for n-D filters," IEEE Trans. Acoustics, Speech and Signal Processing, vol. 27, February, 1979, pp. 1-4.

3. N. K. Bose, "A result on the use of modular methods in multidimensional computations," Archiv für Elektronik und Übertragungstechmik, Band 33, 1979, pp. 213-218.

4. N. K. Bose and K. A. Prabhu, "Two-dimensional discrete Hilbert transform and computational complexity aspects in its implementation," IEEE Trans. Acoustics, Speech, and Signal Processing, vol. 27, Aug. 1979, pp. 356-360.

5. N. K. Bose, "Multivariate polynomial positivity test efficiency improvement," Proc. of the IEEE, vol. 67, Oct. 1979, pp. 1443-1444.

6. K. A. Prabhu and N. K. Bose, "Number of irreducible q-ary polynomials in several variables with prescribed degrees," IEEE Trans. Circuits and Systems, vol. 26, November 1979, pp. 973-975.

7. N. K. Bose and S. Basu, "Theory and recursive computation of 1-D matrix Padé approximants," IEEE Trans. Circuits and Systems, vol. 27, April 1980, pp. 323-325.

8. N. K. Bose and S. Basu, "Two-dimensional matrix Padé approximants: existence, nonuniqueness, and recursive computation," IEEE Trans. Auto. Control, vol. 25, June 1980, pp. 509-514.

9. N. K. Bose and J. P. Guiver, "Multivariate polynomial positivity invariance under coefficient pertubration," IEEE Trans. Acoustics, Speech, and Signal Proc., vol. 28, Dec. 1980, pp. 660-665.

10. S. Basu and N. K. Bose, "Stability of 2-D matrix rational approximants from input data," IEEE Trans. Auto. Control, vol. 26, April 1981, pp. 540-541.

11. J. P. Guiver and N. K. Bose, "On test for zero-sets of multivariate polynomials in noncompact polydomains," Proc. IEEE, vol. 69, April 1981, pp. 467-469.

12. K. A. Prabhu and N. K. Bose, "Impulse response arrays of discrete-space systems over a finite field," IEEE Trans. Acoustics, Speech, and Signal Proc., vol. 30, Feb. 1982, pp. 10-18.

13. J. P. Guiver and N. K. Bose, "Polynomial matrix primitive factorization over arbitrary coefficient field and related results," IEEE Trans. Circuits and Systems, vol. 29, October 1982, pp. 649-657.

14. J. P. Guiver and N. K. Bose, "Strictly Hurwitz property invariance of quartics under coefficient perturbation," IEEE Trans. Auto. Control, vol. 20, January 1983, pp. 106-107.

15. S. Basu and N. K. Bose, "Matrix Stieltjes series and network models," SIAM J. Math. Analysis, vol. 14, March 1983, pp. 209-222.

16. H. M. Valenzuela and N. K. Bose, "Maximally flat rational approximants in multidimensional filter design," Circuits, Systems, and Signal Processing, vol. 2, #1, 1983.

17. J. P. Guiver and N. K. Bose, "Causal and weekly causal 2-D filters with applications in stabilization," chapter 3 of "Recent Advances in Multidimensional Systems" by N. K. Bose (to appear).

BOOKS

1. N. K. Bose, editor "Multidimensional Systems: Theory and Applications," IEEE Press, New York, 1979.

2. N. K. Bose, "Applied Multidimensional Systems Theory," Van Nostrand Reinhold Co., N.Y., 1982.

e. List of the Professional Personnel Associated with the Research Effort

1. K. A. Prabhu

   Completed his Ph.D. dissertation entitled, "Two-dimensional digital filtering in a finite field." Was awarded the Ph.D. degree in Electrical Engineering from the University of Pittsburgh in 1980. Currently employed by Bell Laboratories, Holmdel, New Jersey, 07733.

2. S. Basu

   Completed his M.S. thesis entitled "Theory and application of a direct test procedure for polynomial zeros on polydisc distinguished boundary." Was awarded the M.S. degree in Electrical Engineering from the University of Pittsburgh in 1978.

   Subsequently, completed his Ph.D. thesis entitled, "Multivariable one- and two-dimensional Padé approximation theory and its applications." Was awarded the Ph.D. degree in Electrical Engineering from the University of Pittsburgh in 1980. Currently, serving as an Assistant Professor in the Department of Electrical Engineering, Stevens Institute of Technology, Hoboken, New Jersey, 07030.

3. John P. Guiver

   Completed his Ph.D. dissertation on "Contributions to two-dimensional systems theory." Was awarded the Ph.D. degree in Mathematics from the University of Pittsburgh in 1982. Currently serving as a Guidance and Control Engineer with British Aerospace, at Bristol, England.

4. Hector M. Valenzuela

   Completed his M. S. in December, 1980. Worked on research in the area of

"Maximally flat rational approximants in multidimensional filter design," and used this research to meet the requirements of Ph.D. Preliminary Examination in Electrical Engineering in September, 1981. Was admitted as a candidate for the Ph.D. degree in Electrical Engineering in November, 1982. Is expected to complete all requirements for the Ph.D. degree by October, 1983. His Ph.D. dissertation will be in the area of "Shift-Variant multidimensional systems," which research is a continuation of research efforts in multidimensional linear shift-invariant systems.

5. John Loney

   Worked in the very initial stages of research in rational approximation theory, for a period of one month only.

6. Robert Wen

   Implemented some examples required by the Principal Investigator, on the computer.

7. H. M. Kim

   Beginning his Ph.D. studies in Electrical Engineering following the completion of his M.S. in 1982. Is being trained to serve as a graduate student researcher to the Principal Investigator following the completion of Hector M. Valenzuela's Ph.D. dissertation.

f. Interactions

I. Seminars or talks

1. N. K. Bose, "Test for polynomial zeros and consequences," presented at the International Symp. on Circuits and Systems at New York, May 1978.

2. N. K. Bose, "2-D matrix Padé approximants - existence, nonuniquenss and recursive computation," invited talk at the 4th International Symp. on Mathematical Theory of Networks and Systems, Delft, Holland, on July 3, 1979.

3. N. K. Bose, "Multivariate positivity," invited talk at I.I.T, New Delhi, India, on July 6, 1979.

4. N. K. Bose, "Aspects of 2-D systems theory: stability and positivity," invited talk at Tohoku University, Sendai, Japan, on July 16, 1979.

5. N. K. Bose, "2-D discrete Hilbert transform and computational complexity aspects in its implementation," invited talk in the Special Session on <u>Computational Complexity</u> at the 1979 International Symposium on Circuits and Systems, Tokyo, Japan, on July 17, 1979.

6. N. K. Bose, "Multidimensional Padé approximation theory," invited talk in the Special Session on <u>Distributed and Multivariable Networks</u> in the 1979 Interantional Symposium on Circuits and Systems, Tokyo, Japan on July 18, 1979.

7. N. K. Bose, "Algebraic methods for stability testing," invited talk at the First Acoustics, Speech, and Siganl Processing Workshop on Two-Dimensional Digital Signal Processing at Lawrence Hall of Sciences, Berkeley, California on October 3, 1979.

8. N. K. Bose, "Mathematical realizability theory," invited talk at the Carnegi Mellon University and University of Pittsburgh Joint Colloquium, Department of Mathematics, Pittsburgh, on February 8, 1980.

9. N. K. Bose, "Aspects of two-dimensional digital signal processing," invited talk at the Carnegie-Mellon University Colloquium in Electrical Engineering, Pittsburgh, on March 20, 1980.

10. N. K. Bose, "Finite field methods in multidimensional systems theory," invited talk in a Special Session on Recent Advances in Mathematical System Theory at the International Symposium on Circuits and Systems, Houston, Texas, April 29, 1980.

11. S. Basu and N. K. Bose, "Matrix Stieltjes series and network models," presented by S. Basu at the Midwest Symposium on Circuits and Systems, Toledo, Ohio, on August 5, 1980. Also presented by N. K. Bose as an invited talk at Department of Mathematics, University of Antwerp on June 17, 1982 and at Gesamthochschule Wuppertal on June 24, 1982.

12. K. A. Prabhu and N. K. Bose, "Impulse response arrays of discrete-space systems over a finite field," presented by N. K. Bose at the 1981 International Symposium on Circuits and Systems, Chicago, Illinois, April 28, 1981.

13. N. K. Bose, "Topics in multidimensional systems theory," invited talk at University of Illinois, Chicago, Illinois, on January 21, 1982.

14. N. K. Bose, "Polynomial matrix primitive factorization over arbitrary coefficient field and related results," at Phillips Research Lab., Brussels on June 18, 1982, at Department of Mathematics, Vrije Universiteit, Amsterdam on June 21, 1982, at Department of Mathematics, Technische Hogeschool, Eindhoven on June 23, 1982 and at Institute of

of Automatic Control and Industrial Electronics at ETH, Zurich on June 20, 1982.

15. N. K. Bose "Bidimensional digital filtering over a finite field - latest results," invited seminars at Laboratory on Digital Signal Processing, Swiss Federal Institute, Lausanne, on June 25, 1982 and at Institute of Automatic Control and Industrial Electronics, ETH, Zurich on June 29, 1982.

16. N. K. Bose "Applications of 2-D systems theory: spatio-temporal filtering over finite and infinite fields," invited seminar at Department of Mathematics, Technische Hogeschool, Eindhoven June 22, 1982.

17. N. K. Bose was invited to give four seminars on recent results in multidimensional systems theory including "stabilization of two-dimensional feedback systems by causal and weakly causal compensators," at Department of Electrical Engineering, University of Waterloo on July 12, July 13, July 15 and July 16, 1982.

II. Professional Recognitions and Activities

1. Regular reviewer for Zentralblatt für Mathematik and Mathematical Reviews.

2. Regular reviewer of papers submitted to journals including: Proceedings of IEEE, IEEE Transactions on ASSP, CAS, and Automatic Control, Automatica, Int. J. on Control, J. of the Franklin Institute, and Canadian Electrical Engineering Journal.

3. Elected to be Fellow of IEEE for contributions to multidimensional systems theory and circuits and systems education, effective January 1, 1981.

4. Served as Associate Editor, IEEE Transactions on Circuits and Systems between June 1980 to June 1981.

5. Serving as Chairman of Education Committee, IEEE Society on Circuits and Systems, since 1979.

6. Serving as member of steering committee on Midwest Symp. on Circuits and Systems, since 1969.

7. Invited to chair a Session on "Large-scale dynamical systems," at the First International Conference on Circuits and Computers, Port Chester, New York, on October 2, 1980.

8. Organized a Pre-Symposium Workshop on "Applied Multidimensional Systems Theory," on April 26, 1981 preceding the 1981 International Symposium on Circuits and Systems, held at Chicago, Illinois between April 27, 1981 to April 29, 1981. The workshop was very well attended by participants, both from industries and universities, and was considered to be highly successful.

9. Invited to participate in a workshop on <u>Rational Approximations for Systems</u> at Leuven, Belgium on August 31 - September 1, 1981.

10. Invited to be a member of the Program Committee of the European Conference on Circuit Theory and Design at the 1981 European Conference on Circuit Theory and Design, held at the Hague, Netherlands, August 25-28, 1981.

11. Invited to be a member of the Technical Program Committee, in charge of systems at the International Symposium on Circuits and Systems, to be held at Newport Beach, California, May 2-4, 1983.

12. Reviewer of research proposals submitted to the Engineering Division of National Science Foundation on August 1978, June 1980, and July 1982.

13. Reviewer of proposal submitted to the Division of International programs of National Science Foundation on January 1980 (U.S./Australian Joint Seminar/Workshop) and January 1983 (US-West European Cooperative Science Programs).

14. Reviewer of research proposal submitted to the Air Force.

15. Invited to serve on the Technical Program Committee in the area of Multidimensional Processing at the forthcoming International Conference on Acoustics, Speech and Signal Processing to be held in San Diego, California, March 19 - 21, 1984.

16. Invited to write a monograph on Recent Advances in Multidimensional System Theory by the Managing-Editor, Professor Dr. M. Hazewinkel of Mathematics and Its Applications, D. Reidel Publishing Company.

17. Invited to edit a Special Issue on "Spatio-Temporal Filtering," to be published by Circuits, Systems, and Signal Processing, Birkhäuser Boston Inc.

g. Specific Applications Stemming from Research Report

1. The Padé type of rational approximants occur in the problem of partial realization of systems from prescribed input-output map. These approximants are also known to be closely linked to Prony's method, whose system theoretic applications include its use in the identification of linear dynamic systems from time-domain measurements. Apart from the system theoretic relevance of Padé theory, other engineering applications include time domain design of digital filters and order reduction of control systems. In the problem of digital filter design the coefficients of the power series correspond to the impulse response sequence of the filter. The problem, then, is to find via the Padé method a realizable transfer function or matrix (in the multivariable or multivariable bidimensional cases, which have been the prime targets in the initial stages of this research) from a finite section of the impulse response characteristics. In the context of order reduction of control systems, the objective is to find a transfer function or matrix of lower order than the original transfer function or matrix such that the suitable predetermined number of Markov parameters of the lower order system match with those of the original one.

2. The investigation into the structure and properties or matrix polynomials, orthogonal over a real interval, has been considerably illuminated by the physical insights offered from the well developed theory of passive linear multiport network synthesis, to which the mathematical topic is shown to be closely linked. The developed topic of matrix orthogonal polynomials over a real line is the first comprehensive treatment of the question and its subsequent influence in multichannel continuous system theory (the multi-channel discrete system theory and matrix orthogonal polynomials over an unit circle were topics investigated in 1978 by researchers in U.S.A. and Europe).

3. Since, in physical problems, an approximant is required to be stable, algebraic tests for stability were fully developed. As a result, these algebraic tests for verifying whether or not a multidimensional linear shift-invariant filter is stable are now well established and, if desired, can be implemented with infinite precision, provided adequate computational resources are available. Recognizing the computational chore needed to test for stability, in the 2-D setting, a criterion for stability of a 2-D matrix rational approximant over a Hankel grid was developed, based on the input data. The criterion yields a sufficient condition for BIBO stability.

4. Contributions to the problem of stability invariance under coefficient fluctuation within a symmetric interval centered around their original values are of direct interest when attempting to resolve the multidimensional digital filter stability problem under coefficient fluctuation due to any sources of error-roundoff, quantization etc, as well as in the study of linear active analog filter stability properties when the objective is to determine the allowable ranges of variation of one or more parameters (like gains of op-amps imbedded in the filter), without affecting system stability. Therefore, the foregoing contributions will be useful in the incorporation of nonlinearity in models used in multidimensional filter stability investigations.

5. The results obtained on the zero-sets of multivariate polynomials in non-compact polydomains will find use in multidimensional discrete-continuous stability investigations including, for example, the study of asymptotic stability, independent of delay, in differential equations of the neutral type.

6. The study of the periodicity, autocorrelation and crosscorrelation properties of the matrix impulse response sequence associated with a transfer matrix having coefficients in Z_q, and obtained results on means to construct maximal length matrix impulse response sequences via application of feedback on the state model

constructed from the specified transfer matrix or otherwise, are expected to be useful for simultaneous ranging to several targets, multiple terminal system identification, and possibly code-division multiple access communication systems.

7. A consequence of the primitive factorization algorithm developed is that the gcd extraction algorithm, general factorization theorem, equivalence of minor and factor left (or right) coprimeness concepts in the 2-D case go through with no restriction on the coefficient field K.

8. The primitive factorization theorem enables us to construct the desired factors not only when the elements of a given matrix are in D[w] with D = K[z] but also when D is any Euclidean domain. Let $K_c(z)$ and $K_\sigma(z)$ denote, respectively, the rings of proper real rational functions and stable proper real rational functions. Then, a rational matrix $T(z,w)$ of order $m \times \ell (m \leq \ell)$ with entries in $K_c(z)(w)$ or $K_\sigma(z)(w)$ has a matrix fraction description, which after the application of the primitive factorization theorem can be written in the form,

$$T(z,w) = T_1^{-1}(z,w)\ T_2(z,w)$$

where the matrices $T_1(z,w)$, $T_2(z,w)$ each have entries in $K_c(z)[w]$ or $K_\sigma(z)[w]$ and are relatively left coprime there. It may be possible to set up a synthesis scheme based on these coprime factors, which might yield a realization of lower overall dimension in the dynamic elements z and w than is feasible via Eising's method. It is of interest to note that the lossless positive real property is characterizable by the factors in the matrix fraction description of a square transfer matrix with entries in K(z), where K, here, is the field of real numbers. With the availability of the primitive factorization theorem here, it may be possible to extend these ideas to characterize as well as synthesize 2-D matrices representing various types of systems including 2-D discrete space systems (causal and weakly causal as well as differential dealy systems and lumped-distributed networks).

9. The following result has been proved: A multivariable 2-D plant $P(z,w)$ with a left coprime matrix fraction description, $P(z,w) = D_L^{-1}(z,w)\, N_L(z,w)$ is stabilizable by means of a causal compensator if and only if the m^{th} order minors of $[D_L\ N_L]$ have no common zeros in the closed unit bidisc, U^{-2}. The 2-D setting provides a nice theoretical framework for generalization of these stabilization results to weakly causal systems. Among other results, it has been shown that many causal systems which are not stabilizable by means of a causal compensator can be stabilized by means of a weakly causal compensator. (Here stabilizable is meant in the control sense - i.e. the feedback system is stable). In fact given a scalar plant $\frac{n(z,w)}{d(z,w)}$ with $d(0,0) \neq 0$, $n(0,0) = 0$ one can achieve a structurally stable feedback system by means of a causal compensator if and only if $n(z,w)$ and $d(z,w)$ have no common zeroes in the unit bidisc, whereas if one allows a weakly causal compensator it is only necessary that $n(z,w)$ and $d(z,w)$ have no common zeroes on the distinguished boundary of the unit bidisc. Similar results hold in the multi-input/multi-output or multi-variable case.

10. Explicit conditions required to be satisfied by the numerator and denominator polynomial coefficients of a magnitude-squared rational response function, $|S_{21}(jw_1, jw_2)|^2$, so that it might approximate an ideal low-pass characteristic along any straight line in the plane through (0,0) in a maximally flat manner, have been obtained. The conditions for attenuation along all radial directions through (0,0) is expressed in terms of a positivity of a form in two variables. The technique is easily extended to the multidimensional case and can be used to design filters other than those of the low-pass type. The theory could be used to design lumped-distributed and variable-parameter filters.

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